ROUGHNESS COEFFICIENT IN MOUNTAIN RIVERS

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ABSTRACT: River discharge estimations require an evaluation of the river flow resistance that generally is related with Manning's roughness coefficient. The literature suggests different ways of estimation of Manning's roughness, but in general they can be applied to channels in normal conditions such as uniform flow. But gravel bed rivers have a much larger roughness coefficient than people may think and the literature indicates. The use of a traditional estimation of roughness then would lead us to an underestimation of the roughness, resulting in an overestimate of the flow velocity and an underestimate of the river depth. The purpose of this paper is to recommend formulas to estimate the roughness coefficient for mountain rivers. These empirical formulas were obtained by using data from rivers in the United States, United Kingdom, and Chile. The range of error in the estimation of the roughness using the formulas recommended in this paper has been reduced considerably compared with earlier research. The average error ranged around 6% for rivers with large roughness-scale and 3% for rivers with intermediate roughness-scale. The Froude number and the relative submergence play an important role in the roughness increase in this kinds of channels compare with non-mountain rivers. The data shows that supercritical flow does not occur in this kind of channel.

1.0 INTRODUCTION

It is often necessary to estimate the discharge of mountain rivers where a direct method of measurement cannot be carried out because of the high channel gradients, turbulence, and high flow velocity may make impossible the use of any equipment to measure the flow. In this case empirical estimations can be done, but the main problem is then related to the evaluation of the river flow resistance. Flow resistance is related with the physical shape and bed roughness of a channel, which both control the depth, width, and discharge of the flow in the channel. The three most common resistance coefficient are Manning's (n), Chezy (c), and Darcy-Weisbach (f). These are related to each other by

$$\sqrt{\frac{8}{f}} = \frac{c}{\sqrt{g}} = \frac{R^{\frac{1}{6}}}{n \cdot \sqrt{g}} \qquad (1)$$

where g = acceleration due to gravity; and R = hydraulic radius.

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In this paper the Manning roughness coefficient (n) will be treated, but any of the equations that will be presented can be written in terms of Chezy or Darcy-Weisbach. Manning's resistance coefficient is related to the velocity by:

$$n = \frac{(Sf)^{\frac{1}{2}} R^{\frac{2}{3}}}{V} \qquad (2)$$

Where *Sf* = friction slope (or energy gradient) and *V* = the mean flow velocity .

2.0 FLOW CHARACTERISTICS

According to previous research, mountain river flow can be characterized by the concept of relative submergence (R/D84), or ratio of R, to sediment size, D84. According to this the flow can be characterized from the region of large-scale of roughness (0<R/D84<1), the region of intermediate-scale of roughness (1<R/D84<4) to the region of small-scale of roughness (R/D84 >4). Here R = hydraulic radius and D84 is the size of the river bed material, which is larger than 84% of the material (Bathurst 1985).

3.0 ANALYSIS OF EXISTING EMPIRICAL FLOW RESISTANCE FORMULAS

Strickler (1923) proposed to estimate the Manning roughness coefficient by:

$$n = \frac{(C \cdot Dc)^{\frac{1}{6}}}{21} \qquad (3)$$

Where C = river shape factor and Dc = river critical diameter. There is disagreement among researchers about the value of Dc (Henderson, 1966, Subramaya, 1982, and Madrid-Aris, 1992). Researchers use different material sizes (D50, D84, D90) and coefficients in their empirical formulas. According to Madrid-Aris (1992) it is recommended to use C = 10 for D50 and C = 3.4 for D84 in ranges of relative submergence between 1.0 and 12.5 (1.0 < R/d84 < 12.5).

Limerinos (1970) proposed the estimation of Manning coefficient (*n*) by:

$$n = \frac{0.113 \cdot R^{\frac{1}{6}}}{1.16 + 2 \cdot \log(R/D84)} \qquad (4)$$

Hey (1979) proposed to estimate the Darcy-Weisbach roughness coefficient by a semilogarithmic equation. Thus,

$$\sqrt{\frac{8}{f}} = 5.62 \cdot \log(\frac{a \cdot R}{3.5 \cdot D84}) \qquad a = 11.1 \cdot (\frac{R}{H\max})^{-0.314} \qquad (5)$$

Bathurst (1985) proposed to estimate the Darcy-Weisbach roughness coefficient for mountain rivers by semilogarithmic equation. This formula is similar to Hey formula but considering a average value of parameter a.

$$\sqrt{\frac{8}{f}} = 5.62 \cdot \log(\frac{R}{D84}) + 4.0 \qquad (6)$$

Jarret (1984), proposed an empirical formula obtained using Colorado Rivers data to estimate Manning's roughness coefficient independent of the river bed material.

$$V = 3.17 \cdot R^{0.83} \cdot Sf^{\ 0.12} \quad \text{or} \quad n = 0.32 \cdot Sf^{\ 0.38} \cdot R^{-0.16} \quad \text{in S.I. units} \qquad (7)$$

Where Sf = friction slope. This formula was developed for a large scale of roughness.

Madrid-Aris (1992) proposed 4 empirical formulas based on data from Chilean rivers, for small and intermediate range of roughness , (1<R/D84<12.5),. Two of those formulas are :

$$\sqrt{\frac{8}{f}} = 1.317 \cdot \ln(\frac{R}{D84}) + 5.356 \qquad (8)$$

$$V = 18.15 \cdot R^{0.639} \cdot Sf^{\ 0.461} \quad \text{or} \quad n = 0.055 \cdot R^{0.028} \cdot Sf^{\ 0.039} \quad \text{in S.I. units} \qquad (9)$$

Ugarte and Mendez (1994) proposed an empirical formula using Jarret (1984), Bathurst (1985) and Madrid-Aris (1992) to estimate Darcy Weisbach roughness coefficient for any range of roughness between 0 and 16.

4.0 VALIDATION OF THE EXISTING FLOW RESISTANCE FORMULAS

In order to test the empirical formulas (3,4,5,6,7) and test the applicability of the semilogarithmic resistance law, data were collected from 19 sites in Chilean rivers. A full description of the field survey done is given in Madrid-Aris (1992). The data collected consider small and intermediate scales of roughness (1< R/d84 < 12.5).

First, the Stricker equation (3) was validated in the sense of finding the factor C that minimizes the difference between the roughness estimated and the roughness measured. The results are:

Bed Material	C (shape factor)	General Formula
D90	C = 10.0	$n = 0.070 \cdot D50^{1/6}$
D84	C = 3.4	$n = 0.058 \cdot D84^{1/6}$
D90	C = 2.8	$n = 0.0565 \cdot D90^{1/6}$

Using C = 1 the error obtained was approximately 45% while in the case of using the shape factor indicated the average error was reduced to 14.8% .

Then the Limerinos formula was validated and no change is required to minimize the errors. Validation leads to an average error of 14.5% (see figure 1) in the range of intermediate and small scale of roughness.

Bathurst's formula gives an average error of 22%. Jarret's formula gives an average error of 19%. Madrid-Aris's formulas were obtained using this data and the average error is 13.9% (eq. 7) and 13.3% respectively (see figure 2).

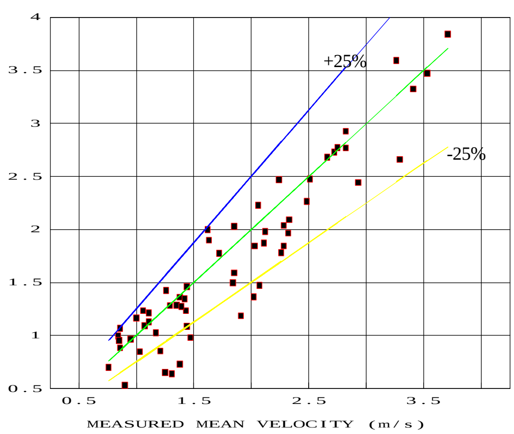


FIG 1.- Predicted mean velocity for Eq. 4 (Limerinos) versus measured mean velocity velocity
(Using Chilean data, Madrid-Aris, 1992).

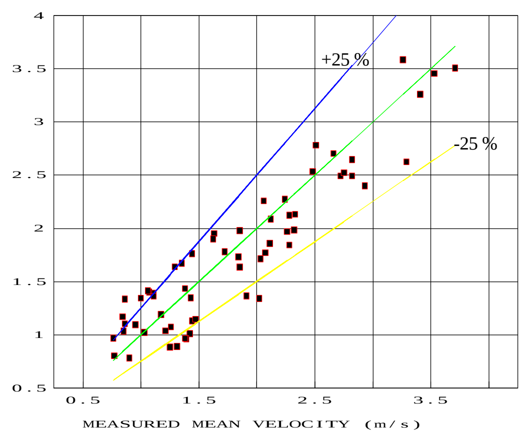


FIG 2.- Predicted mean velocity for Eq. 8 (Madrid-Aris) versus measured mean

(Using Chilean data, Madrid-Aris, 1992).

The aforementioned results suggest that the equations designed explicitly for mountain rivers are not yet well proven, or are restricted to some range of roughness. Semilogarithmic equations work as well as any other (3, 4) in the intermediate and smal range of roughness. Strickler and Limerinos formulas (3,4) do not work at all in the range of large scale of roughness (R/D84 < 1) , average errors are 47% and 63 % respectively using Bathurst and Jarret data.

In general all these formulas based only on the relative submergence (R/D84) do not provide a exact estimation of the flow resistance, with the underestimation or overestimation of the resistance.

5.0 PROBLEM ANALYSIS

The roughness in a mountain river depends on many unexplained factors including turbulence, sediment transport, and water jumps. According to the latest empirical research, there is not an accurate empirical formula to estimate roughness that can be applied to any range of roughness because most of them have been obtained using local river data.
Analyzing the results obtained using different equations, validated with data from Chilean rivers, it is possible to propose the estimation of Manning coefficient (*n*) by:

$$n = nb + \Delta n \qquad (10)$$

Where *nb* is the base value of roughness and it can be obtained the modified Strickler equation by:

$$nb = \frac{(3.4 \cdot Dc)^{\frac{1}{6}}}{21} = \frac{0.183\,(D84)^{\frac{1}{6}}}{\sqrt{g}} \qquad (11)$$

This expression was determined by multiple regression using 62 observations of Chilean rivers. In equation 10, Δn can be estimated by the following adimensional expression:

$$\frac{\sqrt{g}}{(D84)^{\frac{1}{6}}} \cdot \Delta n = f\left(Sf, \frac{R}{D84}, Fd\right) \qquad (12)$$

where *Fd* is the Froude number, *Sf* = the friction slope and *R/D84* is the relative submergence.

5.1 Flow Resistance Equation for Large-Scale of Roughness (0<R/D84<1)

On the basis of the data published by Bathurst (1985) related to U.K. rivers and the data published by Jarret (1984) of Colorado rivers, the following expression was determined to estimate the Manning resistance coefficient.

$$n = \left[0.183 + \ln\left(\frac{1.7462 \cdot Sf^{0.1581}}{Fd^{0.2631}}\right)\right] \cdot \frac{(D84)^{\frac{1}{6}}}{\sqrt{g}} \qquad \text{in S.I. units} \qquad (13)$$

Where the bed material size *(D84)* is considered in meters. Using this empirical formula (14) to estimate the mean velocity, the mean error is about 5.7%, the greatest error is equal to 21%. It is important to mention that the data used to estimate this formula considers a range of small flows, the highest being equal to 10.3 m^3/s.

5.1 Flow Resistance Equation for Intermediate-Scale of Roughness (1<R/D84<12.5)

On the basis of the data published by Madrid-Aris (1992) related to 62 measures collected in 19 Chilean rivers with relative scales of roughness between 1 and 12.5, the following expressions were determined to estimate the Manning resistance coefficient.

$$n = [0.183 + \ln(\frac{1.3014 \cdot Sf^{0.0785} \cdot (\frac{R}{D84})^{0.0211}}{Fd^{0.1705}})] \cdot \frac{(D84)^{\frac{1}{6}}}{\sqrt{g}} \quad \text{in S.I. units} \quad (14)$$

$$n = [0.219 + \ln(\frac{1.3259 \cdot Sf^{0.0932} \cdot (\frac{R}{D50})^{0.0260}}{Fd^{0.2054}})] \cdot \frac{(D50)^{\frac{1}{6}}}{\sqrt{g}} \quad \text{in S.I. units} \quad (15)$$

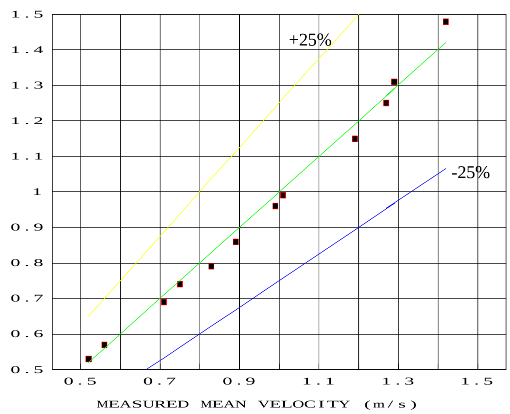
The bed material size is considered in meters. Using this empirical formula to estimate the mean velocity, the mean error is only 2.2% , the greatest error is equal to 9.9% (figure 4).

The range of flow of the data used in the determination of formulas 14 and 15 ranges between 371 m^3/s and 2.71 m^3/s. It is important to mention that the same data was used to estimate the semilogarithmic equation 8 and with this new approach the error was reduced from 13.9% to only 2.2%. This leads us to believe that this approach better explains the phenomenon of roughness in mountain rivers than does the semilogarithmic approach.

6.0 VALIDATION OF THE FORMULAS PROPOSED

It was impossible to validate equation 13 with Chilean data because there are no measurement sites in places with high scales of roughness. But the validation was made using Boulder Creek River data (see Thorne and Zevenbergen 1985). The average error in the estimation of the mean velocity is 2.7 %., the greatest error is equal to 4.8% (figure 3).

In order to test the validity of equations 14 and 15 data was used from the U.K. and Colorado. The average error in the estimation of the mean velocity is 6.3%., the greatest error is 37% . It is important to mention that the results obtained are quite accurate considering that the formula was developed using a much high range of flow.



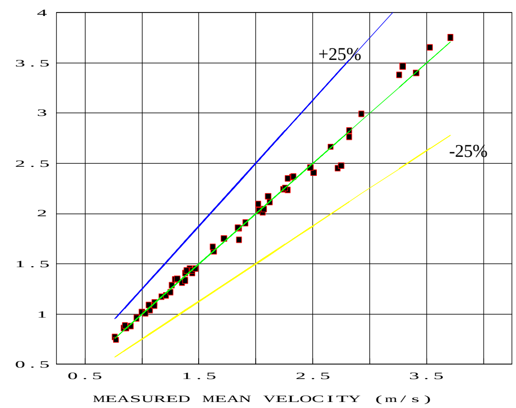


FIG 3.- Predicted mean Velocity for Eq. 13
4
(Ugarte-Madrid) versus measured mean
velocity (Thorne and Zevenbergen data)
1992).

FIG 4.- Predicted mean Velocity for Eq.
(Ugarte-Madrid) versus measured mean
velocity (Chilean data-Madrid-Aris

7.0 CONCLUSIONS

1. Validation of the Stickler formula is required before using it for roughness estimation in mountain rivers. Good results can be obtained with this simple formula using the factor of shape (C) recommended in the range of small and intermediate range of roughness.

2. It is not recommended to use traditional formulas such as Strickler or Limerinos in the range of large scale of roughness.

3. The semilogarithmic resistance formulas to estimate the Darcy-Weisbach roughness coefficient do not explain very well the phenomenon of roughness in mountain rivers.

4. The roughness in mountain rivers varies in a complicated way, depending on relative submergence, Froude number and other characteristic factors relating to the flow. These factors should be investigated to account for energy losses due to turbulence, sediment transport, and hydraulics jumps.

5. The application of formula 14 or 15 is recommended, especially in the 1 to 12.5 range of roughness and for river slopes ranging from 0.2% to 4%. According to the results obtained, this model explains accurately the phenomenon of mountain river roughness .

6. It is necessary to continue the research in the range of large-scale of roughness (R/d84%<1) but equation 13 can be considered a good preliminary result because results obtained are far better than previous research. Future research should consider a broader range of flow for this range of roughness.

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